

**FACULTY OF SCIENCE**  
**B.A./B.Sc. (CBCS) V – Semester (Backlog) Examination, May/June 2024**

Subject : Mathematics  
 Paper – V : Linear Algebra

Max. Marks: 80

Time: 3 Hours

PART – A

(8x4=32 Marks)

Note : Answer any Eight questions.

1. Show that the set of all matrices  $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \mid a, b, c, d \in R \right\}$  is a subspace of the vector space  $M_{2 \times 2}(R)$  of all  $2 \times 2$  matrices with real entries.

2. If  $\begin{bmatrix} 3 \\ -4 \end{bmatrix} = a \begin{bmatrix} 3 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ , then find  $\begin{bmatrix} a \\ b \end{bmatrix}$ .

3. Show that the set  $S = \{1, x+1, x^2+2\}$  is a basis of the vector space of all polynomials  $p_2(R)$  of degree less than or equal to 2.

4. Find the rank of the matrix  $A = \begin{bmatrix} 1 & 3 & 5 & 6 \\ 3 & 4 & 6 & 8 \\ 2 & 6 & 10 & 12 \\ 4 & 7 & 11 & 14 \end{bmatrix}$ .

5. If the null space of a  $8 \times 5$  matrix A is 2-dimensional, then find the dimension of the row space of A.

6. Find the eigen vectors of the matrix  $A = \begin{bmatrix} 1 & -1 \\ 5 & 7 \end{bmatrix}$ .

7. Show that the mapping defined by  $T: P_2(R) \rightarrow R^2$  defined by  $T(p) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}$  is a linear transformation. (Here  $p(t) = a_0 + a_1t + a_2t^2$ ,  $a_0, a_1, a_2 \in R$ )

8. Find the eigen values of the matrix  $A = \begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$ .

9. Find the matrix of the linear transformation  $T: R^3 \rightarrow R^3$  defined by  $T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ a+b \\ b+c \end{bmatrix}$  with respect to

the basis  $B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$  of the vector space  $R^3(R)$ .

10. If  $u = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$  and  $v = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$  then find  $u \cdot v$  and  $\|u+v\|$ .



11. If  $y = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$  and  $u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , then find the orthogonal projection of  $y$  onto  $u$ .

12. Let  $W$  be a subspace of the vector space  $R^n(R)$ . Show that the set  $W^\perp = \{x \in R^n \mid x \cdot u = 0 \text{ for all } u \in W\}$  is a subspace of  $R^n$ .

### PART - B

Note : Answer all the questions.

(4 x 12 = 48 Marks)

13. (a) Find bases of the Null space and the Column space of the matrix

$$A = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

OR

(b) (i) Let  $B = \{b_1, b_2, \dots, b_n\}$  be basis of vector space  $V$ . Then show that for each  $x \in V$  there exists a unique set of scalars  $c_1, c_2, \dots, c_n$  such that  $x = c_1 b_1 + c_2 b_2 + \dots + c_n b_n$ .

(ii) Show that the set  $S = \left\{ \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\}$  is a linearly dependent set in the vector space  $R^2(R)$ .

14. (a) State and prove rank theorem.

OR

(b) Find the eigen vectors of the matrix  $A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ .

15. (a) Show that an  $n \times n$  matrix  $A$  is diagonalizable if and only if  $A$  has  $n$  linearly independent eigen vectors.

OR

(b) Construct the general solution of  $x' = Ax$  where  $A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ .

16. (a) (i) If  $u$  and  $v$  are vectors in the vector space  $R^n$ , show that

$$\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2).$$

(ii) If  $S = \{u_1, u_2, \dots, u_p\}$  is an orthogonal set of non zero vectors in  $R^n$ , then show that  $S$  is linearly independent.

OR

(b) Using Gram Schmidt process, construct an orthogonal basis for the subspace  $W$  of  $R^4$

spanned by the vectors  $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 0 \end{bmatrix}$  and  $v_3 = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ .

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**FACULTY OF SCIENCE**  
**B.A./B.Sc. (CBCS) V- Semester Examination, December 2023/January 2024**

**Subject: Mathematics**  
**Paper-V: Linear Algebra**

Time: 3 Hours

Max. Marks: 80

**PART - A**

**Note: Answer any eight questions.**

(8x4= 32 Marks)

1. Prove that the intersection of two subspaces is again a subspace.
2. Verify whether the set  $\{(1,1,2) (2,2,4) (1,3,4)\}$  is linearly independent.
3. If a vector space  $V$  has a basis set  $B = \{b_1, b_2, \dots, b_n\}$  then prove that any set in  $V$  containing more than  $n$  vectors must be linearly dependent.
4. Find the eigen values of  $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ .
5. Find the smallest possible dimensions of  $\text{nul } A$ , given that  $A$  is of  $3 \times 7$  matrix.
6. Prove that eigen values of matrix  $A$  and it's transpose  $A^T$  are the same.
7. Mention under what condition the given matrix is diagonalizable.
8. If  $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$  then find eigen values and a basis for each eigen space in  $C^2$ .
9. Suppose  $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$  then find  $A^4$  given that  $A = PDP^{-1}$ , where  $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$  and  $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ .
10. Show that the set vectors  $\{u_1, u_2, u_3\}$ , where  $u_1 = \left[\frac{3}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{3}{\sqrt{11}}\right]^T$ ,  $u_2 = \left[\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]^T$ ,  $u_3 = \left[\frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}}\right]^T$  are orthogonal.
11. Prove that two vectors  $u$  and  $v$  are orthogonal if and only if  $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ .
12. In an inner product space, prove that any orthogonal set of non-zero vectors is linearly independent.

**PART - B**

**Note: Answer all the questions.**

(4x12= 48 Marks)

13. (a) (i) Prove that the column space of an  $m \times n$  matrix  $A$  is a subspace of  $\mathbb{R}^m$ .

(ii) If  $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ , if column space of  $A$  and the null space of  $A$  are subspaces of  $\mathbb{R}^k$ . Then find the value of  $k$ .

(OR)

- (b) (i) Find the dimension of the null space and the column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

- (ii) If a vector space  $V$  has a basis of  $n$  vectors, then prove that every basis of  $V$  must contain exactly  $n$  vectors.



14. (a) (i) Find bases for the row space, the column space and the null space of the matrix

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 1 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}.$$

(OR)

(b) State and prove Rank theorem.

15. (a) Diagonalize the matrix  $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$  if possible.

(OR)

(b) Suppose  $A = \begin{bmatrix} 0.5 & -0.6 \\ 0.75 & 1.1 \end{bmatrix}$ , then find the eigen values of  $A$  and find a basis for each eigen space.

16. (a) State and prove orthogonal decomposition theorem.

(OR)

(b) Explain the Gram-Schmidt process.

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## FACULTY OF SCIENCE

B.A. / B.Sc. (CBCS) V – Semester Examination, December 2022 / January 2023

Subject: Mathematics

Paper – V : Linear Algebra

Time: 3 Hours

Max. Marks: 80

## PART – A

Note: Answer any eight questions.

(8 x 4 = 32 Marks)

1. Define a vector space and give an example of vector space.
2. Prove that the intersection of two subspaces is again a subspace.
3. If  $A = \begin{bmatrix} 6 & -4 \\ -3 & 2 \\ -9 & 6 \end{bmatrix}$  then find Null space of  $A$ .
4. Find the eigen values of  $A = \begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix}$ .
5. Find rank of a matrix having order  $4 \times 7$  with 4-dimensional null space.
6. If  $\lambda$  is an eigen value of an invertible matrix  $A$ , then prove that  $\frac{1}{\lambda}$  is an eigen value of the matrix  $A^{-1}$ .
7. Is every matrix diagonalizable? Mention the condition for the given matrix to be diagonalizable.
8. Find the eigen values and a basis for each eigen space in  $C^2$  for  $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ .
9. Prove that an  $n \times n$  matrix with  $n$  distinct eigen values is diagonalizable.
10. If  $u = [2, -5, -1]^T$  and  $v = [3, 2, -3]^T$  then find the inner product of  $u$  and  $v$ .
11. If  $u, v$  are two vectors. Then prove that two vectors,  $u, v$  are orthogonal if and only if  $\|u - v\|^2 = \|u\|^2 + \|v\|^2$ .
12. Prove that, in an inner product space, any orthogonal set of non-zero vectors is linearly independent.

## PART – B

Note: Answer all the questions.

(4 x 12 = 48 Marks)

13. (a) (i) Given  $V_1$  and  $V_2$  in a vector space  $V$ . Let  $H = \text{span} \{V_1, V_2\}$  then show that  $H$  is a subspace of  $V$ .

(ii) Prove that the null space of an  $m \times n$  matrix  $A$  is a subspace of  $R^n$ .

(OR)

(b)(i) Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

(ii) Let  $\beta = \{b_1, b_2, \dots, b_n\}$  be a basis for a vector space  $V$ . Then prove that for each  $x \in V$  there exists a unique set of scalars  $c_1, c_2, \dots, c_n$  such that  $x = c_1 b_1 + c_2 b_2 + \dots + c_n b_n$ .



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14. (a) (i) State and prove Rank theorem.

(ii) Let  $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$   $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$   $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$   $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$  and consider the bases for  $R^2$  given by  $B = [b_1, b_2]$  and  $C = [c_1, c_2]$  then find the change of coordinates matrix from  $B$  to  $C$ .

(OR)

(b) (i) If  $V_1, V_2, \dots, V_r$  are eigen vectors that correspond to distinct eigen values  $\lambda_1, \lambda_2, \dots, \lambda_r$  of an  $m \times n$  matrix  $A$ , then prove that the set  $\{V_1, V_2, \dots, V_r\}$  is linearly independent.

(ii) Find the characteristic equation of  $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

15. (a) Diagonalize the matrix  $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$  if possible.

(OR)

(b) Suppose  $B = [b_1, b_2]$  is a basis for  $V$  and  $C = [c_1, c_2, c_3]$  is a basis for  $W$ . Let  $T: V \rightarrow W$  be a linear transformation with the property that  $T(b_1) = 3c_1 - 2c_2 + 5c_3$  and  $T(b_2) = 4c_1 + 7c_2 - c_3$ . Then find the matrix  $M$  for  $T$  relative to  $B$  and  $C$ .

16. (a) Suppose  $A$  is  $m \times n$  matrix. Then prove that the orthogonal complement of the row space of  $A$  is the null space of  $A$  and the orthogonal complement of the column space of  $A$  is the null space of  $A^T$ .

(OR)

(b) Explain the Gram Schmidt Process.



**FACULTY OF SCIENCE**  
**B.Sc./ BA V Semester (CBCS) Examination, March 2022**

**Subject: Mathematics**  
**Paper - V : Linear Algebra**

**Max. Marks: 80**

**Time: 3 Hours**

**PART - A**

**(8 x 4 = 32 Marks)**

**Note: Answer any eight questions.**

1. Determine whether the Set  $S = \{v_1, v_2, v_3\}$  is a basis of  $\mathbb{R}^3$ , where

$$v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix} \quad v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix} \quad v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}.$$

2. Prove that intersection of two subspaces is again a subspace.

3. Find the dimension of the subspace H spanned by  $\begin{bmatrix} 1 \\ -5 \end{bmatrix}$ ,  $\begin{bmatrix} -2 \\ 10 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ 15 \end{bmatrix}$ .

4. If a  $7 \times 5$  matrix A has rank 2, Find  $\dim \text{Nul } A$ ,

5. If  $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$  an eigen vector of  $\begin{bmatrix} -4 & 3 & 3 \\ 2 & -3 & -2 \\ 1 & 0 & -2 \end{bmatrix}$  then, find eigen value.

6. Find the characteristic polynomial of  $A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$

7. Show that an  $n \times n$  matrix with  $n$  distinct eigen values is diagonalizable.

8. Let  $T: V \rightarrow W$  be a linear transformation with  $T(b_1) = 3c_1 - 2c_2 + 5c_3$  and  $T(b_2) = 4c_1 + 7c_2 - c_3$ . Find the matrix M for T relative to bases  $B = \{b_1, b_2\}$  and  $C = \{c_1, c_2, c_3\}$  for vector spaces V and W.

9. Find the complex eigen values of  $A = \begin{bmatrix} 0 & 5 \\ -2 & 2 \end{bmatrix}$ .

10. Find a unit vector in the direction of  $(1, -2, 2, 0)$ .

11. Determine if  $\{u_1, u_2, u_3\}$  is an orthogonal set, where  $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ ,  $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ ,  $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$

12. Let  $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$  and  $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ . Find the orthogonal projection of y onto u.



## PART - B

(4 x 12 = 48 Marks)

Note: Answer any four questions.

13. State and prove spanning set theorem.

14. Define nul space and find basis for the nul space of matrix

$$A = \begin{bmatrix} 1 & 1 & -2 & 1 & 5 \\ 0 & 1 & 8 & -1 & -2 \\ 0 & 1 & 0 & -1 & 14 \end{bmatrix}$$

15. State and prove rank theorem.

16. Find eigen values and eigen vectors of  $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ .

17. Compute  $A^6$ , where  $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$  using  $A = PDP^{-1}$

18. Construct general solution of  $x' = Ax$  where  $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$ .

19. If  $S = \{u_1, u_2, \dots, u_p\}$  is orthogonal set of non zero vectors in  $\mathbb{R}^n$ , then prove that  $S$  is linearly independent and hence is a basis for subspace spanned by  $S$ .

20. Let  $W$  be the subspace spanned by the set  $S = \{x_1, x_2, x_3\}$  where

$$x_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -5 \\ 1 \\ 5 \\ -7 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 8 \end{bmatrix}. \quad \text{Now Construct an orthogonal basis for } W.$$

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Time: 2 Hours

## PART – A

(4 x 5 = 20 Marks)

Note: Answer any four questions.

1 Prove that intersection of a subspace is again a subspace.

2 Determine if  $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$  is in col A, where  $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ .3 Determine if  $\{v_1, v_2, v_3\}$  is basis for  $\mathbb{R}^3$ , where  $v_1 = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$ ,  $v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$ .4 Let  $B = \{b_1, b_2\}$  and  $C = \{c_1, c_2\}$  be bases for a vector space V and suppose  $b_1 = 6c_1 - 2c_2$  and  $b_2 = 9c_1 - 4c_2$ . Then find change of coordinate matrix B to C.5 Find the complex Eigen values of the matrix  $A = \begin{bmatrix} 3 & -3 \\ 3 & 3 \end{bmatrix}$ .6 Compute  $A^4$  where  $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ ,  $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  and  $A = PDP^{-1}$ .7 Compute  $\|u+v\|$  where  $u = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}$ ,  $v = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$ .8 Find a unit vector in the direction of  $v = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$ .

## PART – B

Note: Answer any two questions.

(2 x 20 = 40 Marks)

9 Define Null space and find spanning set for the Null space of given matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

10 If a vector space V has a basis  $\beta = \{b_1, b_2, \dots, b_n\}$  then show that any set in V containing more than n vectors must be linear dependent.



11 State and prove that Rank theorem. Also find rank A, where  $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$ .

12 Prove  $\lambda = 4$  is an Eigen value of  $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$  and find the corresponding Eigen vector and characteristic equation of A.

13 Diagonalize  $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ , if possible.

14 (i) Find the orthogonal projection of  $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$  onto the line through  $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$  and the origin.

(ii) Determine if the set  $\{u, v, w\}$  is orthogonal set. Given  $u = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$ ,  $v = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix}$ .

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**FACULTY OF SCIENCE**  
**B.Sc. V Semester (CBCS) Examination, November / December 2021**

**Subject: MATHEMATICS**  
**Paper: V – Linear Algebra**

Max. Marks: 60

Time: 2 Hours

**PART – A****(4 x 5 = 20 Marks)****Note: Answer any four questions.**

- 1 Prove that  $H = \left\{ \begin{bmatrix} s \\ t \\ 0 \end{bmatrix} : s, t \text{ are real} \right\}$  is a subspace of  $\mathbb{R}^3$ .
- 2 Prove that Null A is subspace of  $\mathbb{R}^n$ .
- 3 Determine if  $\{v_1, v_2, v_3\}$  is L.D or L.I, where  $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ .
- 4 Find the dimension of the subspace H of  $\mathbb{R}^2$  spanned by  $\begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 15 \end{bmatrix}$ .
- 5 Let  $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$  and  $\beta = \{b_1, b_2\}$ . Find the coordinate vector  $[x]_\beta$  of x relative to  $\beta$ .
- 6 Diagonalize A, where  $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$  and  $v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$  are Eigen vectors of A.
- 7 Compute distance between  $u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$  and  $v = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$ .
- 8 Suppose y is orthogonal to vectors u and v, then show that y is orthogonal to  $u + v$ .

**PART – B****(2 x 20 = 40 Marks)****Note: Answer any two questions.**

- 9 Show that  $H = \text{span} \{v_1, v_2\}$  is a subspace of a vector space V and determine if  $W = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$  is subspace spanned by  $\{v_1, v_2, v_3\}$  where  $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ .
- 10 Let  $\beta = \{b_1, b_2, \dots, b_n\}$  be basis for vector space V. Then show that the coordinate mapping  $x \rightarrow [x]_\beta$  is 1-1 linear transformation from V on to  $\mathbb{R}^n$ .



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11 If two matrices A and B are row equivalent, then show that their row spaces are the same. Also find  $\dim \text{Row } A$ , where  $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$ .

12 Find eigen values and eigen vectors of  $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ .

13 Diagonalize  $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ , if possible.

14 (i) Prove that two vectors  $u$  and  $v$  are orthogonal iff  $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ .

(ii) Determine if the set  $\{u, v\}$  is orthogonal. If so, find the orthonormal set. Given

$$u = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, v = \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

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